ROBUSTNESS OF MULTI-LEVEL PUBLIC TRANSPORT NETWORKS: A METHODOLOGY TO QUANTIFY ROBUSTNESS FROM A PASSENGER PERSPECTIVE

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ABSTRACT

Despite the importance of robust public transport networks, this topic has not been considered from a full passenger perspective yet in scientific literature and practice. To our best knowledge, this study is the first in which both exposure to large, non-recurrent disturbances and impact of these disturbances are analysed in a systematic and realistic way. Contrary to single-level network perspectives, we considered the integrated, total multi-level public transport network which remains available when a disturbance occurs. We developed a new methodology to identify the most vulnerable links in the multi-level public transport network and to quantify the societal costs of non-robustness of these vulnerable links. Besides, applying our methodology enables quantification of the robustness benefits of robustness measures, next to the costs of such measures. Therefore, our methodology can support and rationalize the decision-making process of public transport operators and authorities regarding the implementation of different robustness measures.

Keywords: disturbances, multi-level public transport networks, passenger perspective, value of robustness, vulnerability

1. INTRODUCTION

The operation of public transport services without disturbances is considered a key quality aspect of public transport (PT) by passengers (Golob *et al.*, 1972; Van Oort, 2011). Therefore it is important to get insight in the frequency with which disturbances occur in public transport, and the impact of these disturbances on passengers. Despite the fact that passengers perceive a punctual PT operation without disturbances as important, this topic is not fully considered from a passenger perspective in scientific literature and practice yet. Three conclusions based on literature study support this proposition.

First, current research mainly focuses on service reliability, which is defined as the matching degree between actual and scheduled PT operations in relation to recurrent, often small disturbances which do not influence infrastructure availability. Research on improvements of reliability of single-level PT networks is amongst others conducted by Hollander (2006) and Van Oort and Van Nes (2009) (see Figure 1 upper left quadrant). In these studies, reliability is only considered for a certain PT network level (for example: only for the regional or urban PT network) or for a PT network operated by a single operator. Examples of measures to improve reliability of single-level PT networks on a strategic, tactical and operational level can be found in Vromans *et al.* (2006), Delgado (2009), Furth (2009), Corman *et al.* (2010), Van Oort *et al.* (2010), Van Oort and Van Nes (2010) and Xuan *et al.* (2011). Besides considering reliability of single-level PT networks, also research on reliability of multi-level PT networks is conducted where interactions between different network levels are considered (for example Rietveld *et al.*, 2001; Lee *et al.*, 2014) (see Figure 1 upper right quadrant). Such multi-level approach of reliability allows for example the incorporation of the consequences of a delay on the train network for transfers to a lower-level bus connection.

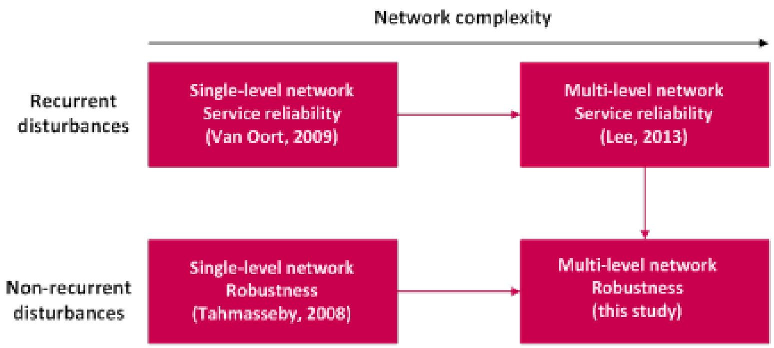

Figure 1. Relevance of study focusing on robustness of multi-level public transport networks, including examples of references in other quadrants

Second, studies to PT robustness, which is related to deviations of PT services from schedule caused by non-recurrent, often – but not necessary – large disturbances which reduce infrastructure availability, focus on single-level PT networks only (Figure 1 quadrant left under). In scientific literature, different definitions for robustness are used (for example Ziha, 2000; Holmgren, 2007; Van Nes *et al.*, 2007; Tahmasseby, 2009; Korteweg and Rienstra, 2010; Savelberg and Bakker, 2010; Snelder, 2010; Immers *et al.*, 2011; Parbo *et al.*, 2013; Dewilde *et al.*, 2014). An extensive review of definitions and indicators for reliability can be found in Nicholson *et al.* (2003). In our study we use a definition of robustness which is, contrary to reliability, only related to non-recurrent disturbances: *'Robustness is the extent to which a multi-level PT network is able to maintain the function it was originally designed for in case of non-recurrent disturbances which reduce infrastructure availability'*. The function of a PT network is defined as *'providing connections to passengers within the expected travel time, against expected travel costs and with the expected travel comfort'*. Reliability and robustness are distinguished from each other based on the criterion whether a disturbance reduces infrastructure availability on a certain link or not. Non-recurrent disturbances which reduce infrastructure availability lead to adjustments in the supplied PT services. This, in turn, leads to negative passenger effects. From the passenger perspective we adopt in this study, especially for large disturbances it is not sufficient to consider only travel time effects of non-recurrent disturbances. It is important to consider all effects these disturbances have on passengers:

effects on connectivity, travel time, travel costs and travel comfort. Examples of studies to robustness of single-level PT networks are Goverde (2005), Kroon *et al.* (2008), Tahmasseby (2008), Cicerone *et al.* (2009), Fischetti *et al.* (2009), Schöbel and Katz (2009) and Corman *et al.* (2014). These studies analyse robustness separately for each PT network on a certain functional network level (single-level perspective), or for a PT network operated by a specific operator (single-operator perspective). Both in science and in practice, it is often not or hardly considered how a certain network level, or PT network operated by operator X_1, can function as backup in case a disturbance occurs on another network level operated by operator X_2.

However, when using a passenger perspective in case of large disturbances, it is important to consider the integrated multi-level PT network which remains available for passengers (Figure 1 quadrant right under). This means that the PT networks on all functional network levels, operated by different operators, should be considered in an integrated way. In their total door-to-door trip, passengers usually use PT services on different network levels, often operated by different PT operators. For example, in the period 2006-2009 on average 89.8% of the trips having train as main mode in the Netherlands can be considered multimodal (Van Nes *et al.*, 2014). In case each PT operator only optimizes the part of the network she operates, it is likely that different optimized subnetworks lead to a suboptimal total network from a passenger perspective, since interactions between network levels are ignored. In case of large disturbances, this leads to suboptimal rescheduling from a passenger perspective because network levels of other operators, which might have potential to function as backup, are not considered. Possible powerful measures on network level X_1 which can improve robustness for passengers because of disturbances on network level X_2 are then not investigated. This also means that there is no full and no realistic evaluation of passenger impacts of disturbances, since passengers are able to consider the total available multi-level PT network in case of disturbances. We therefore conclude that currently no full passenger perspective is adopted regarding PT robustness in science and practice.

Third, in discussions about robustness usually only the costs of robustness measures are known (for example: the costs of a new switch). However, the societal costs of disturbances, and the benefits of these robustness measures are unknown and not quantified until now. However, it is important to have insight in both the societal costs *and* benefits of each robustness measure in order to have full information in the decision-making process regarding the implementation of such measure.

Our study fills in these knowledge gaps (see Figure 1). We developed a methodology to identify the most vulnerable links in a multi-level public transport network. Based on this method we are able to evaluate the robustness of these identified vulnerable links given the total multi-level network available. This allows the quantification of societal costs of non-robustness for passengers in a more realistic way, since passengers also consider the multi-level network when looking for route alternatives in case of a disturbance. This methodology is applied in a case study in the Randstad Zuidvleugel area in the Netherlands. In this case study we compare the robustness of different network levels. We use the integrated multi-level PT network to design measures to improve robustness of identified most vulnerable links and to quantify the robustness benefits of these measures. Applying our methodology therefore supports decision-making regarding the implementation of measures aiming to improve robustness of public transport networks.

The structure of this paper is as follows. Chapter 2 discusses the methodology developed to identify vulnerable links and to quantify robustness of these links. In chapter 3 we show the results of applying this methodology in a case study. We finish the paper in chapter 4 by formulating conclusions and recommendations for further research.

2. METHODOLOGY

2.1 Robustness and vulnerability

Vulnerability and robustness are inversely related: a network with 0% vulnerability yields 100%

robustness, and the other way around (Tahmasseby, 2009; Snelder, 2010). The higher the vulnerability – the sensitivity of PT services to major disturbances in terms of connectivity, travel time, travel costs and comfort – the less a PT network is able to provide passengers connections within the expected travel time, against expected travel costs and with the expected travel comfort. Assume a multi-level PT network represented by a digraph $G(V_n, E_n)$ with nodes $v_n \in V_n$ and links $e_n \in E_n$ on PT network level n. From a passenger perspective, the vulnerability of a link $e_n \in E_n$ depends on the extent to which that link e_n is exposed to non-recurrent disturbances δ and the consequences of these disturbances δ for passengers given the total PT network N available. The vulnerability c_{e_n} of a link e_n can therefore be expressed by formula (1).

$$c_{e_n} = \sum_{\delta_n} E\left(f_{e,\delta_n}\right) * E\left(\tau_{e,\delta_n}\right) * \Delta W_{e,\delta_n,\tau} \tag{1}$$

The exposure of a link e_n to large disturbances is the product of the frequency f_{e,δ_n} with which different disturbance types δ occur on that link and the duration τ_{e,δ_n} of each disturbance. Both the frequency with which disturbance types occur and the duration of each disturbance are probabilistic variables, which are independent from each other for each disturbance type δ. Therefore this leads to a multiplication of the expected number of disturbances $E\left(f_{e,\delta_n}\right)$ occurring within a certain time window and the expected duration of each disturbance $E\left(\tau_{e,\delta_n}\right)$, with both $f_{e,\delta_n}$and τ_{e,δ_n} being random variables. $\Delta W_{e,\delta_n,\tau}$ represents the difference in total monetized societal costs for all passengers travelling over all OD-pairs affected by that specific disturbance δ_n between the specific disruption scenario and the undisturbed situation.

Traditionally, especially for PT networks link vulnerability c_{e_n} is only assessed based on the impact of a disturbance. This can be explained by the limited historic data available about the frequency with which different disturbance types δ_n occur on each PT network level n and their related duration τ_{δ_n}. This means that exposure to disturbances is not considered explicitly when determining link vulnerability. Instead, a conditional vulnerability is applied which calculates the impact of a disturbances given the fact that a certain disturbance has occurred, see formula (2). When applying formula (2), links where disturbances have the most negative impact on passengers' travel time, costs and comfort are listed most vulnerable, even if the frequency with which these disturbances occur would be very low. Instead, formula (1) incorporates exposure to disturbances explicitly: links where the combination of exposure to disturbances and the impact of these disturbances is very high are considered most vulnerable.

$$c_{e_n} = \Delta W_{e,\delta_n,\tau} | \delta_n \tag{2}$$

For this study a unique dataset is analysed, containing realization data about the frequency and duration of different types of disturbances δ_n on different PT network levels n (national / interregional / regional / agglomeration / urban level) and for different PT modes (train / metro / light rail / tram / bus) operated by different PT operators in the Netherlands. This enables us to incorporate exposure to disturbances on each PT network level explicitly when considering network robustness. For both the frequency and duration of each disturbance type δ_n is statistically tested whether the empirical data fit a theoretical probability distribution function. By distribution fitting, parameter values f_{δ_n} and τ_{δ_n} are estimated for the probability distribution functions for each disturbance δ_n. Because especially the frequency f_{δ_n} with which some disturbances occur can be influenced by the weather, it is tested whether significant seasonal differences exist in average frequency of each disturbance δ_n. In that case separate parameters are estimated for different seasons. In Yap (2014) an extensive description of this data analysis can be found.

All disturbances δ_n are categorized based on two dimensions: the impact of a disturbance on infrastructure availability and the impact on PT demand. Some disturbances usually lead to a partial unavailability of a link (like a tram breakdown), whereas other disturbances lead to a complete unavailability of a link (like a train-car collision on a level crossing). Disturbances with a low level of predictability (an unplanned disturbance) will usually not or hardly affected PT demand on affected OD-pairs, since passengers had no *a priori* knowledge about the occurrence of such disturbance. However, planned disturbances with a high level of predictability (like announced large track works) can reduce PT demand *a priori*. Passengers on affected OD-pairs can then decide to change mode or destination choice, or to cancel the trip at all. Because $\Delta W_{e,\delta_n,\tau}$ will be different in case PT services on a certain link are partial or completely cancelled, or in case PT demand is affected on beforehand or not, it is necessary to distinguish different disruption scenarios S based on these two dimensions for which the link vulnerability c_{S,e_n} can be calculated.

2.2 Identification of vulnerable links

When aiming to improve PT network robustness, it is important to identify which links are most vulnerable in the multi-level PT network. In scientific literature two different approaches are applied to identify the most vulnerable network links (Knoop *et al.*, 2012). The first approach uses full computation methods. In these methods, disturbances are simulated on each link $e \in E$ of the network separately to evaluate its vulnerability relative to other links. The advantage of such methods is obviously their completeness, since vulnerability of the complete set of links E can be assessed and compared. The largest disadvantage is that these approaches can be very time consuming. In the second approach criteria are specified to pre-select a smaller number of most vulnerable links in a network. Disturbances are only simulated on these selected links in a second step. This approach overcomes the disadvantage of very long computation times of full-computation methods. However, since pre-selection criteria are used to identify a short-list of vulnerable links, there is no guarantee that indeed the most vulnerable links are remaining after the pre-selection phase.

Since real-life, complex multi-level PT networks as we consider in this study are usually modelled by a very large number of links, computation times become unacceptable long when all relevant disruption scenarios would be simulated on each link separately. Therefore, it is necessary to apply a method to pre-select most vulnerable links. In scientific literature various criteria can be found to pre-select the most vulnerable links of road networks. However, there is very limited literature where pre-selection criteria for public transport networks are specified. Only some examples can be found, e.g. in Cats and Jenelius (2014) using a dynamic vulnerability analysis, and in Bell (2003) and Zhang *et al.* (2010) using game theory. Therefore we developed a new methodology to identify most vulnerable links in multi-level PT networks. This methodology is based on existing methodologies developed to identify vulnerable links for road networks (Jenelius *et al.*, 2006; Li, 2008; Tampère *et al.* 2008; Immers *et al.*, 2011; Knoop *et al.*, 2012) and is adjusted based on PT network characteristics.

We can formulate four important conclusions when analysing the suitability of road network pre-selection criteria for identification of vulnerable links in multi-level PT networks. First, criteria which consider the probability on disturbances on road networks calculate this probability on a link level. For PT networks it makes more sense to calculate incident probabilities per link *segment*. PT operators usually apply standard procedures in case of disturbances: for each location in the network a disruption scenario specifies how PT services are adjusted in case of partial or complete track unavailability. Because rescheduling possibilities for PT services depend on the availability of switches, turning loops, station capacity etc., these procedures will be exactly equal for adjacent links with no switches or other rescheduling possibilities in between them. Such procedures are therefore designed per link segment – a set of adjacent links taken together by the PT operator for which one standard disruption scenario applies – instead of per link. Second, for road networks some criteria only consider the impact of a disturbance, whereas other criteria consider both the probability on a disturbance and its impact explicitly. Criteria which only consider the incident impact, implicitly assume an equal probability on disturbances on each link. When incident probabilities are considered for road networks, often one generic predictor (e.g. link length) is used to distinguish incident probabilities for different links. However, Yap (2014) shows that for identified disturbances δ_n on a multi-level PT network different predictors (like link segment length, vehicle-kilometres per link segment) should be used to distinguish incident probabilities for different link segments. Also, it is clearly shown that probabilities on a certain disturbance type δ are different on different PT network levels, given the different characteristics of these network levels. This shows that it is not sufficient to assume an equal probability on disturbances for all links in a multi-level PT network. Pre-selection criteria for multi-level PT networks should therefore consider both the probability on disturbances (using different predictors for different disturbances δ and different parameter values f_{δ_n} for different network levels) and the impact of a disturbance on passengers explicitly. Third, in road networks some ratio between traffic volume and capacity (like the Incident Impact Factor or V/C ratio) is used as proxy for the impact of a disturbance. Since the real incident impact on travel time, costs and comfort $\Delta W_{e,\delta_n,\tau}$ can only be derived after simulation of disturbances using a full-computation method, a proxy for this impact has to be used in the identification phase. In PT networks the relation between volume and capacity is less relevant when approximating the consequences of a disturbance, since on PT networks very limited congestion occurs between PT vehicles – even in case of disturbances – compared to congestion between vehicles on road networks. The impact of a disturbance in PT networks is mainly related to the number of passengers affected, instead of the V/C ratio of PT vehicles on a certain link. For example, a single-track local train line can have a very high V/C ratio if there are limited possibilities for trains to pass each other, whereas a very busy four-track train line might have a lower V/C ratio. Therefore, the passenger flow on affected links is a better proxy to represent the impact of an incident. Fourth, some pre-selection criteria for road networks only focus on the impact of a disturbance on the considered link e itself, whereas other criteria also consider spillback effects to adjacent links j or gridlock effects. For road networks it is clear that disturbances can have spillback effects to other links. However, in PT networks spillback effects occur differently compared to road networks. Given the limited congestion between PT vehicles, there are no or only limited direct spillback effects to PT vehicles on adjacent link segments in case of disturbances. However, PT services on other link segments $js \neq es$ in the network can certainly be affected by a disturbance on link segment es. As explained, PT operators apply standard disruption procedures. In these procedures PT lines can be cut into two parts, shortened, rerouted over an alternative track or cancelled. For example, if a PT line L is cancelled because of a disturbance on link segment es, passengers travelling over other link segments $js \in L$ are clearly affected. This means that PT services on a certain link segment es can be affected because of a first-order effect – a disturbance occurring on that link segment es itself – and because of a second-order effect. This second-order effect is relevant in case a disturbance occurs on another link segment js, leading to disruption measures taken by the PT operator or infrastructure manager which also affect PT services on the considered link segment es. Except during the transition phase between regular PT operations and the disruption scenario, the spillback effect on PT networks can be considered more static compared to the dynamic spillback effects occurring on road networks. For PT networks, gridlock effects are hardly of relevance.

Based on these formulated conclusions we propose a new methodology to identify the most vulnerable links in multi-level PT networks (Figure 2), for which the next pre-selection criteria I^1 to I^5 are specified.

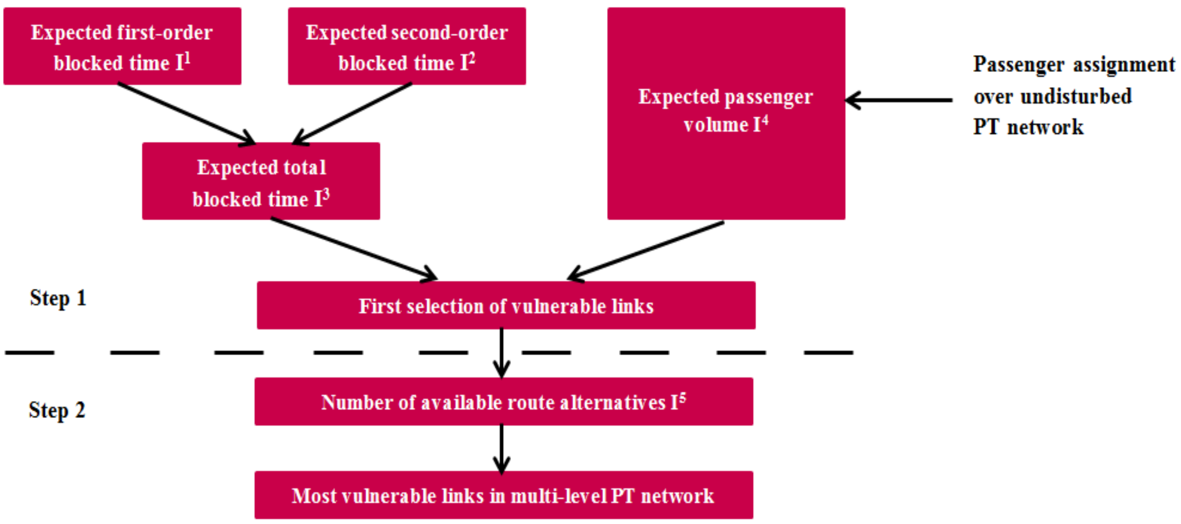

Figure 2. Stepwise methodology to identify vulnerable links in multi-level PT networks

Step 1: calculate I^1, I^2, I^3 and I^4 and make a first selection of vulnerable links

$$I_{es}^1 = \sum_{pr} \sum_{\delta,pr} \sum_{w} f^*{}_{\delta_n prw} * \frac{x_{pr,es}}{x_{pr,n}} * \tau^*{}_{\delta_n w} \qquad \forall \;\; es \in E \tag{3}$$

I_{es}^1 reflects the first-order effect: the expected time that a certain link segment es_n is blocked within a certain time interval because of non-recurrent disturbances occurring on that link segment itself. This equals the product of the average frequency $f^*{}_{\delta_n prw}$ with which disturbance type δ_n occurs per time period on network level n in season w and the average duration $\tau^*{}_{\delta_n w}$ of each disturbance δ_n in season w. For each δ_n a predictor pr is determined which allows transformation of the average frequency with which δ_n occurs per time period on the whole considered network level n (which is known from historic data) to the average frequency per link segment es. This transformation is based on the ratio between the value of this predictor $x_{pr,es}$ on link segment es and the value $x_{pr,n}$ on the total network level n. For this criterion only the average frequency f^* and average duration τ^* are used. Since in that case no Monte Carlo simulation is required to draw values from the identified distribution functions, we can reduce computation times.

$$I_{es}^2 = \sum_{js} \sum_{pr} \sum_{\delta,pr} \sum_{w} f^*{}_{\delta_n prw} * \frac{x_{pr,es}}{x_{pr,n}} * \tau^*{}_{\delta_n w} \qquad \forall \;\; es \in E \tag{4}$$

I_{es}^2 reflects the second-order effect: the expected time that a certain link segment es_n is blocked within a certain time interval because of non-recurrent disturbances occurring on other link segment js, leading to measures taken by PT operators which also influence PT operations on the considered link segment es. In this study we used the procedures as taken by PT operators in the Netherlands in reality in case of disturbances, in order to determine which other link segments $js \neq es$ affect PT services on the considered link segment es in case of disturbances.

$$I_{es}^3 = I_{es}^1 + I_{es}^2 \qquad \forall \;\; es \in E \tag{5}$$

I_{es}^3, being the sum of the first-order and second-order effects, expresses the expected total time a link segment es is blocked within a certain time interval because of non-recurrent disturbances.

$$I_e^4 = q_e \qquad \forall \;\; e \in E \tag{6}$$

Where I_{es}^3 considers the link segment incident probability explicitly, I_e^4 expresses the proxy for the impact of a disturbance based on number of passengers travelling over the considered link e. This value can be determined after performing an undisturbed passenger assignment, showing the passenger volume what would travel over a certain link e in case no disturbances would occur. Because passenger volume can differ over different links $e \in es$, this value is expressed for each link e separately. For all considered links of the multi-level PT network the values of I_{es}^3 and I_e^4 can be plotted against each other. Links with the highest value for $I^3|I_n^4$, or the other way around, appear on the Pareto frontier in this plot. By selecting all links which are plotted on or nearby the Pareto frontier, we can make a first selection of vulnerable links based on these pre-selection criteria. Adjacent links in the network which all appear on the Pareto frontier can be taken together as one link segment.

Step 2: assess I^5 and make a final selection of vulnerable links

$$I_e^5 = \mathit{risk\ by\ limited\ availability\ of\ alternative\ routes}$$

For each vulnerable link (segment) on the Pareto frontier the number of available alternative routes I_e^5 can be assessed, considering the total multi-level PT network N which is available. Similar as for road network links, the number of available alternative routes is often determined qualitatively based on an expert judgment, since the number of available routes differs per OD-pair. Links for which hardly any route alternatives are available in the multi-level PT network are classified as most vulnerable, which leads to a final list of identified most vulnerable links.

2.3 Evaluation and quantification of link non-robustness

When the most vulnerable links of the multi-level PT network are identified, (non-)robustness of these links can be evaluated and quantified. Evaluation of robustness from a passenger perspective can be done by applying formula (1), which explicitly considers both the exposure to disturbances and the impact of these disturbances given the total multi-level PT network N available. As explained in chapter 2.1, different disruption scenarios S can be distinguished for each link based on the impact of a disturbance δ_n on infrastructure availability and PT demand. Given a chosen time horizon for which link robustness is evaluated, Monte Carlo simulation is used to generate disturbances δ_n with a certain duration τ_{δ_n} for all distinguished disruption scenarios. Based on the estimated parameters for frequency and duration of $\delta_{n,w}$, values are drawn from the identified theoretical distribution functions. In the simulation model the PT network and PT services are adjusted according to each disruption scenario S, based on which a new passenger assignment can be performed. The total monetized societal costs $\Delta W_{e,\delta_n,\tau}$ for all passengers travelling over all OD-pairs can then be compared between the undisturbed situation and the specific disruption scenario S.

$$C_t = \sum_{o=1}^{n} \sum_{d=1}^{n} \left(\alpha_a t_a + \alpha_w \sum_{x=1}^{n_t+1} t_{w,x} + \alpha_{in} \sum_{y=1}^{n_t+1} t_{in,y} + \alpha_{n_t} n_t + \alpha_t \sum_{z=1}^{n_t} t_{t,z} + \alpha_e t_e\right) * VoT \quad (7)$$

Formula (7) expresses the calculation of the perceived, monetized travel time effects C_t given a network modelled with n origins O and n destinations D. The different travel time components – access time from origin to a PT stop t_a, waiting time t_w before boarding each PT service (the number of waiting moments x equals the number of transfers $n_t + 1$), in-vehicle time t_{in} for each PT service (the number of used PT services y equals the number of transfers $n_t + 1$), the number of transfers n_t, transfer walking time t_t for each transfer walk z, and the egress time from the PT stop to final destination t_e – are all multiplied by their corresponding weight α as experienced by passengers and monetized using the Value of Time (VoT) (see for the applied Dutch values Bovy and Hoogendoorn-Lanser, 2005 and Warffemius, 2013). Here we used a fixed VoT, independent from the amount of delay on a certain OD-pair. Besides travel time effects, also the effects of disturbances on travel costs C_c are evaluated. Incorporating C_c is especially important when considering disturbances in the context of multi-level PT networks. In case of disturbances passengers often have to take a longer alternative route, sometimes thereby using PT services of another PT operator. In the evaluation of link non-robustness, also the societal costs because of reduced travel comfort are quantified for seated passengers $C_{comf,seat}$ and standing passengers $C_{comf,stand}$, respectively. This is of relevance, since especially during disturbances the load factor on remaining alternative routes in the PT network can increase substantially, thereby reducing passengers' comfort. Also the societal costs of non-facilitated demand C_{non-f} are quantified, since it is possible that during a disturbance passenger volume on a link of a certain alternative route exceeds the total supplied link capacity (seated plus standing capacity). By applying the rule of half on the generalized travel costs for each affected OD-pair, cancellation costs C_{cancel} are quantified for the percentage of travelers which cancel their PT trip *a priori* in case of a disturbance with a high level of predictability. For a more detailed explanation of the quantification of C_c, $C_{comf,seat}$, $C_{comf,stand}$, C_{non-f} and C_{cancel} we refer to Yap (2014).

$$\Delta W_{e,\delta_n,\tau} = \Delta C_t + \Delta C_c + \Delta C_{comf,seat} + \Delta C_{comf,stand} + \Delta C_{non-f} + \Delta C_{cancel} \quad (8)$$

Formula (8) shows all the components based on which the total monetized societal costs $\Delta W_{e,\delta_n,\tau}$ because of a disturbance are calculated. When we consider all distinguished disruption scenarios S specified for a certain link e, we can adjust formula (1) in order to calculate the societal costs of non-robustness of link e within a specified time horizon:

$$c_{e_n} = \sum_S \sum_{\delta_n} E\left(f_{e,\delta_n,S}\right) * E\left(\tau_{e,\delta_n,S}\right) * \Delta W_{e,\delta_n,\tau,S} \quad (9)$$

3. RESULTS CASE STUDY

3.1 Case study network

The developed methodology for identification of the most vulnerable links and evaluation of link robustness is applied in a case study to the Randstad Zuidvleugel, the southern part of the most important economic area of the Netherlands (2.2 million inhabitants). This area is selected because of its relatively high PT network density with PT services on different network levels. The interactions between these network levels are especially interesting when considering multi-level PT networks. The PT network is modelled as supernetwork in a high level of detail with the transport planning software OmniTRANS with 5.791 zones, 106.000 nodes $v \in V_n$ and 116.000 links $e \in E_n$. For important PT lines L the seat capacity and crush capacity are specified. A frequency-based network representation is applied, meaning that waiting time for a PT line L is assumed to be half of the interarrival time between two PT vehicles of that line L. Although a schedule-based representation is more realistic, this requires more detailed model input and increases computation times for passenger assignment substantially. Besides, because of the relatively high frequency of PT lines in the Randstad, the differences in waiting time between a frequency-based and schedule-based network representation remain limited.

In our model four different time periods are distinguished: morning peak 7-8am, morning peak 8-9am, evening peak 4-6pm and the remaining hours of the work day. Especially during the morning peak PT demand is not uniformly distributed over the two hours of the morning peak in the Netherlands (CBS, 2013). Because we consider societal costs of crowding and non-facilitated demand explicitly, assuming a uniformly distributed PT demand would lead to a biased quantification of these costs. Therefore, the morning peak is split in two separate periods 7–8am and 8–9am with separate OD-matrices. The Zenith algorithm is applied for performing a passenger assignment in the undisturbed situation and for distinguished disruption scenarios S (Brands *et al.*, 2013). Despite the mentioned importance of comfort and crowding effects, these aspects are not incorporated in the generalized cost function used for the assignment. This is because especially during unexpected disturbances passengers do not know the crowding level of PT services on alternative routes on beforehand, and often do not have many route choices. Therefore we do not expect that crowding level is dominant as component of the generalized cost function on which passengers base their route choice during disturbances. Besides, incorporating the capacity of PT lines in the assignment would lead to an iterative, capacity-constrained assignment, which increases computation times substantially. The perceived disutility because of discomfort on the chosen route is however incorporated afterwards in the evaluation of link non-robustness by $C_{comf,seat}$ and $C_{comf,stand}$.

3.2 Identification of vulnerable links in the Randstad Zuidvleugel network

By using the historic dataset of realized disturbances on the network levels of different PT operators in the Netherlands as input for calculating the first-order, second-order and total blocked time, vulnerable links could be identified for the case study network. Figure 3 shows the expected first-order, second-order and total exposure to non-recurrent disturbances per year for link segments on the metro / light rail network level. Figure 4 shows the expected total exposure to non-recurrent disturbances per year for link segments on the train, metro / light rail and tram network. The metro and light rail network level are taken together, since these modes operate on the same functional network level. Based on Figure 3 and Figure 4 we can formulate the next four conclusions.

1. First, it is important to incorporate second-order spillback effects when calculating the expected total time a link segment is exposed to large disturbances. Figure 3 clearly shows that the expected total time a link segment es_n is blocked is heavily influenced by disturbances occurring on other link segments $js_n \neq es_n$. Not considering these second-order effects would lead to substantial overestimation of link segment robustness.
2. Second, it becomes clear that the expected total blocked time of light rail link segments near The Hague (triangular dots in Figure 3 up to nr. 283) is substantially larger compared to link

segments of the Rotterdam metro network (triangular dots in Figure 3 from nr. 283). This can be explained because the switch density on the Rotterdam metro network is considerably higher compared to the light rail network near The Hague. Disturbances therefore remain more local, reducing their second-order effect to other link segments. In the Rotterdam metro network, there are switches available near almost every metro station. This means that when a disturbance occurs on a certain metro link segment, thereby blocking the link in either one or both directions, metro services are cut in two separate parts by the operator. The second-order effect then equals the first-order effect, since disturbances occurring on link segment es_{n1} in direction 1 will only affect services on the exact same link segment in the other direction 2 es_{n2} as second-order effect.

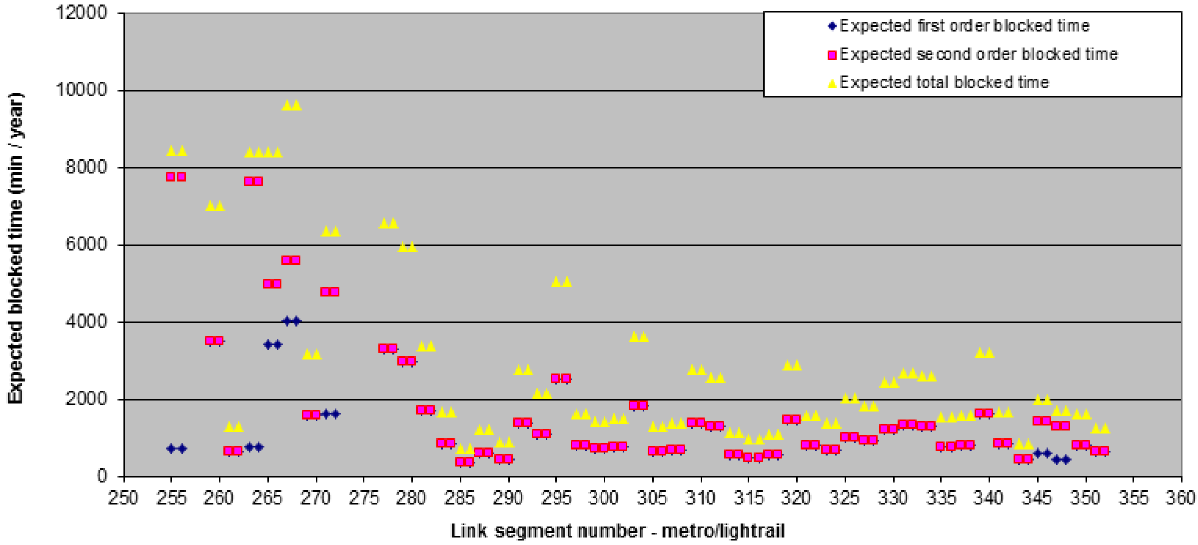

Figure 3. Expected first-order, second-order and total exposure per link segment of the light rail / metro case study network (link segments to nr. 283: The Hague; link segments from nr. 283: Rotterdam): blue dots can be exactly equal to the pink dots in some cases

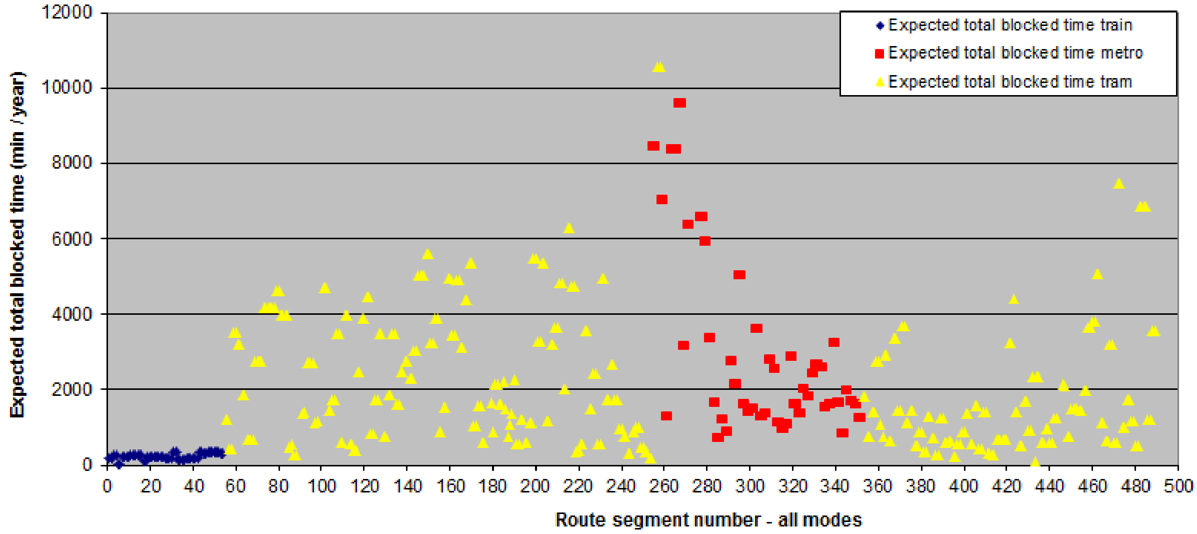

Figure 4. Expected total exposure to disturbances per link segment of the multi-level case study network (triangular left: tram network The Hague; triangular right: tram network Rotterdam)

3. Third, Figure 4 shows that train link segments are relatively robust against exposure to disturbances compared to metro / light rail and tram link segments. Possible explanations for this are the own right of way for trains, the availability of a signaling system to prevent train-train collisions and the relatively low train intensity on train links compared to the intensity on metro, light rail or tram links.

4. Fourth, in general the link segments of the tram network of The Hague (triangular dots left in Figure 4) are more vulnerable to exposure to disturbances compared to link segments of the Rotterdam tram network (triangular dots right in Figure 4). This can partly be explained because in general more parallel (sometimes unused) tram tracks are available in Rotterdam, which can function as backup in case of disturbances and reduce second-order effects. The triangular outlier in the middle of Figure 4 shows the specific link segment Ternoot – Laan van NOI of the tram network of The Hague. This link is located directly before/after the light rail route Laan van NOI – Zoetermeer / Rotterdam, without intermediate rescheduling possibilities. Therefore, second-order effects are relatively large on this link segment. A disturbance on the light rail network often also influences PT services on this tram link segment.

In Figure 5 the results on pre-selection criteria I^3 (expected total exposure to disturbances per year) and I^4 (expected passenger volume) are plotted against each other for each link. Based on this figure we stress the importance of using pre-selection criteria in a methodology to identify vulnerable links in a multi-level PT network which capture *both* exposure to disturbances and the impact of each disturbance explicitly. Figure 5 clearly illustrates that there are link segments on the Pareto frontier of which the impact of a disturbance is expected to be relatively low, but which are very vulnerable because of relatively heavy exposure to disturbances (see for example the most right triangular dots in Figure 5). If only the impact of a disturbance would be considered, only the busiest links of the train network would be identified as most vulnerable. However, given the Pareto frontier where incident probability and impact are both considered, we conclude that there is no network level or mode which clearly contains most vulnerable links. Links on or nearby the Pareto frontier are from the train, metro / light rail and tram network. Train links are especially vulnerable because of the expected large impact of disturbances, whereas metro and tram links are mainly vulnerable when a combination of relatively heavy exposure to disturbances and a relatively large number of affected passengers is expected.

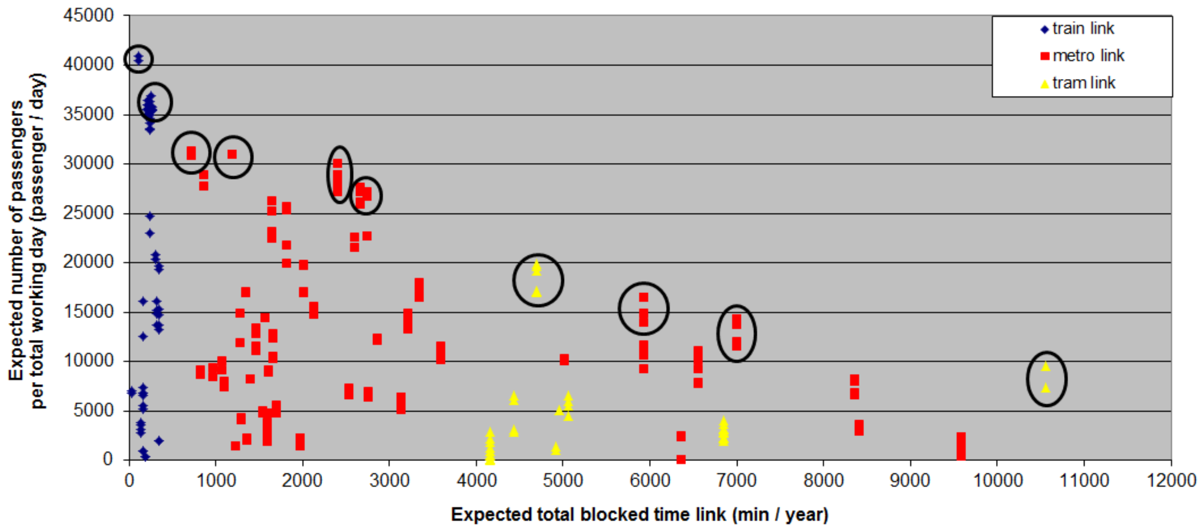

Figure 5. Vulnerability of links of the multi-level case study network by plotting I^3 against I^4

The availability of alternative routes (I^5) is assessed qualitatively for all link segments on the Pareto front in a second step (see Yap, 2014). This leads to the following selection of most vulnerable link segments for this case study network:

- Delft – Schiedam (train).
- Switches Gerdesiaweg / Voorschoterlaan – Kralingse Zoom (metro).
- Brouwersgracht – The Hague Central Station (tram tunnel) (tram).
- Rodenrijs – Melanchtonweg (light rail).
- Laan van NOI – Forepark (light rail).

The developed methodology to evaluate and quantify link robustness is applied to the light rail segment Laan van NOI – Forepark for illustration purposes. PT services on this link segment are operated by two different operators together: the HTM and RET.

3.3 Evaluation and quantification of robustness of link segment Laan van NOI – Forepark

Disturbances are generated using Monte Carlo simulation for a time horizon of 10 years. Based on this simulation we can conclude that during 10 years the light rail segment Laan van NOI – Forepark is blocked for 964 hours. Assuming on average 18 hours PT operation per day, this means that in 1.5% of the time PT services on that link segment are blocked because of large disturbances. In case this link segment is blocked, the model shows which alternative routes in the multi-level PT network are used. Link segment vulnerability c_{es_n} is calculated using formula (9), where $\Delta W_{es,\delta_n,\tau}$ is calculated using formula (8). In the current situation the expected total societal costs of disturbances on this link segment in 10 years equal €4.3 million. This value expresses the costs of non-robustness of the analysed link segment. Figure 6 (left) shows that additional transfers and their related waiting time and transfer time are the most important contributors to these societal costs. During these 10 years 787 disturbances on this link segment are expected according to our simulation results. This means that expected average societal costs per disturbance equal €5.4 thousand.

In this study we also designed and evaluated a measure aiming to improve robustness of this link segment. Since many affected passengers use the local train connection between Zoetermeer, Ypenburg and The Hague as backup during disturbances, we propose a temporary increase in frequency on this connection. We investigated adding two temporary stops for intercity train services operating on this parallel train track at two local train stops, Zoetermeer and The Hague Ypenburg, *only* in case PT operations on the light rail segment Laan van NOI – Forepark are disturbed. In that case, the frequency of (stopping) train services between Zoetermeer, Ypenburg and The Hague is doubled during disturbances. This improves transfer possibilities between network levels and improves the backup function of the train network for the disturbed light rail network. Disadvantage however is that the travel time for through travelers in the intercity service increases with ≈5 minutes.

After generating disturbances using a pseudo-random generator, we can quantify the robustness effects of this measure (see Figure 6 right). Total societal costs of non-robustness after 10 years now equal €3.9 million. This means that this measure reduces the costs of non-robustness of this link segment by 8%, therefore having a positive Net Present Value. The expected average societal costs per disturbance now equal €5.0 thousand. This measure especially reduces waiting time substantially, at cost of an increase in total in-vehicle time. However, monetized benefits from waiting time reduction outweigh the monetized costs of additional in-vehicle time.

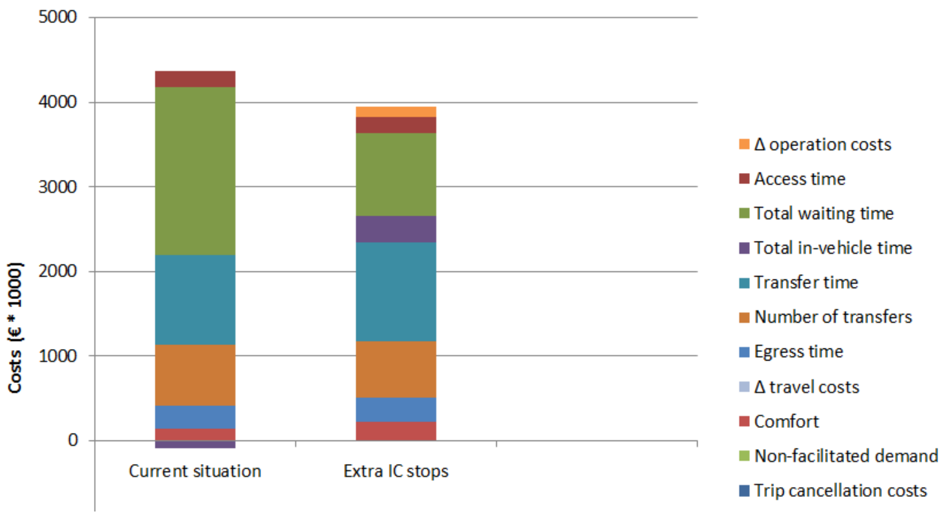

Figure 6. Societal costs of non-robustness of link segment Laan van NOI – Forepark in the current situation (left) and for the proposed measure (right)

4. CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusion and discussion

Despite the importance of robust public transport networks, this topic has not been considered from a full passenger perspective yet in scientific literature and practice. To our best knowledge, this study is the first in which both exposure to large, non-recurrent disturbances and impact of these disturbances are analysed in a systematic and realistic way. Contrary to single-level network perspectives, we considered the integrated, total multi-level PT network which remains available after the occurrence of a certain disturbance. We developed a new methodology to identify most vulnerable links in the multi-level PT network. The case study results show the importance of incorporating exposure to disturbances explicitly in this methodology, since only considering the impact of disturbances would lead to a clearly different list of vulnerable links. We also stress the importance of taking into account second-order spillback effects in this methodology when calculating total link segment exposure to disturbances. Not considering second-order exposure to disturbances can lead to substantial overestimations of link robustness.

Regarding measures aiming to improve robustness, currently only the costs of such robustness measures are known. Based on our methodology we are able to evaluate and quantify the societal costs of disturbances. Besides, applying our methodology enables the quantification of the part of these societal costs which can be reduced by a certain robustness measure. This allows us to express robustness benefits of a certain measure in monetary terms and compare these to the required costs of that measure, thereby quantifying the value of robustness. Therefore, our methodology can support the decision-making process of public transport operators and authorities regarding the implementation of different robustness measures.

It is important to realize that the topic of robustness should always be considered in a trade-off with other aspects. Some robustness measures (like the construction of additional switches) can on the one hand reduce the societal costs of a disturbance, *if* a disturbance occurs, but on the other hand increase the frequency with which disturbances occur. Other measures can reduce the impact of a disturbance for affected passengers, while increasing travel time for other groups of passengers. Some measures might be able to improve robustness substantially on the one hand, but require large investments on the other hand. The result of these trade-offs will be different for different locations in the network and depends on the frequency with which disturbances occur, the impact of disturbances, the number of passengers affected by the disturbance and the extent to which alternative routes are available in the multi-level PT network. Applying the methodology we developed allows decision-makers to get insight in these trade-offs for each specific location. Also, with our methodology all aspects relevant in such trade-off can be expressed in the same, monetary units. Therefore, our methodology helps to support and rationalize the decision-making process regarding public transport robustness measures.

In our case study we quantified the robustness benefits of one specific measure, where the train network functions as backup for a vulnerable link segment of the light rail network. Regarding this evaluation and quantification we can formulate some points for discussion. First, for a successful implementation of measures using the multi-level PT network it is important to consider the distribution of financial and societal costs and benefits over stakeholders involved. Most costs of the proposed measure are for the Dutch train operator NS, because of additional timetable hours their trains have to run and additional travel time for train passengers, although the disturbances occur on the network operated by the HTM and RET. To implement this measure successfully, it seems likely that (financial) incentives have to be provided to the Dutch Railways by PT authorities or the PT operators HTM and RET. Second, we did not quantify the network wide effects for train passengers because of the increased travel time in intercity services, for example whether connections later on the track are missed. We however did check that sufficient buffer time between conflicting trains is available in the timetable on the specific train track in case running time of intercity trains would be extended by 5 minutes in our measure. Third, for a successful implementation of the proposed measure it is required that the RET and HTM provide passengers information about the temporary doubled frequency of train services stopping at the stations

Zoetermeer and The Hague Ypenburg, in order to improve the backup function of the train network. Else, passengers will not adjust their route choice. Fourth, rescheduling costs and costs for recovery of the planned timetable, vehicle and personnel circulation are not considered in our study. Incorporating this would further increase the financial and societal costs of disturbances. Therefore, the societal costs of disturbances as calculated in this paper can be considered as lower bound.

4.2 Recommendations

We formulate four recommendations for further research. First, we recommend validation of our developed methodology to identify vulnerable links in multi-level PT networks. For a test network, the overlap should then be investigated between the set of links indicated as most vulnerable based on our developed methodology and after a complete evaluation of robustness of all links using a full-computation method. Although our methodology is based on validated methodologies applied to road networks, validation can be used for further fine-tuning of this methodology. Second, we recommend investigating the formalization of the second step of our developed methodology to identify vulnerable links. In our study we assessed the number of available alternative routes for vulnerable links qualitatively based on expert judgment. By quantification of this step, our methodology can be further improved. For example, for each OD-pair affected by a disturbance on a certain link, the number of feasible route alternatives and their remaining capacity could be calculated by applying route choice set criteria (see for example Fiorenzo-Catalano, 2007). Third, we recommend to incorporate dynamic en-route route choice in the disturbed passenger assignment based on travel information available to passengers (see for example Van der Hurk *et al.*, 2012). For all our assignments we only considered pre-trip route choice, assuming full information about a disturbance during the whole trip. This shows the potential of the multi-level PT network to function as backup in case of a disturbance. However, in reality disturbances are dynamic and there is not always full information available about the disturbance. Therefore it would be interesting to incorporate the dynamics of disturbances and the role of information provided to passengers, combined with en-route route choice, in the assignment. Fourth, it should be mentioned that a multi-level approach regarding PT robustness is rather complex regarding the collection of revealed data about disturbances occurring on the networks of multiple PT operators, assignment calculation times, and implementation of measures which exceed the borders of the network of a certain PT operator. We therefore recommend PT operators to adopt a real passenger perspective regarding robustness, where the passenger gets priority over the borders of a concession area. In the end, passengers base their mode choice on the total door-to-door trip. Improving robustness of the total multi-level PT network will therefore positively affect PT demand.

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